Edexcel IAL Physics A-level

Topic 4.3: Further Mechanics

Notes

4.3 - Further Mechanics

4.3.81 - Impulse

Newton's 2nd law states $F = ma$, therefore, $F = \frac{\Delta(mv)}{\Delta t}$ as $a = \frac{\Delta v}{\Delta t}$.

You can rearrange the equation above to get:

$$F\Delta t = \Delta(mv)$$

$$F\Delta t = \Delta p$$

Where **F** is a **constant** force, **t** is the **impact time** and **p** is momentum.

$F\Delta t$ is known as impulse, and **impulse** is the change in momentum as demonstrated in the equation above.

Below is an example question about impulse.

A ball is hit with a baseball bat with a force of 100 N, with an impact time of 0.5 s. What is the change in momentum of the ball?

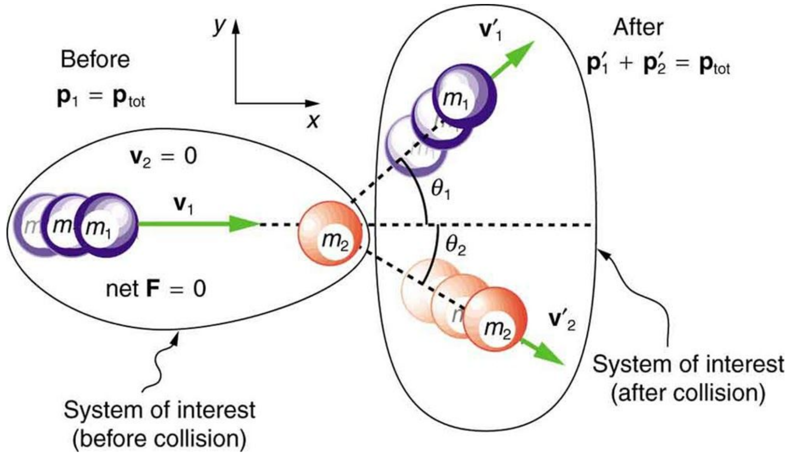
To find the impulse we must use the equation $F\Delta t = \Delta p$.
Change in momentum = 100×0.5 = **50 kgm/s**

4.3.82 - Conservation of linear momentum in two dimensions

Momentum is always conserved in any interaction where no external forces act, which means the momentum before an event (e.g a collision) is equal to the momentum after. It is also important to note that momentum can be conserved differently along different dimensions.

The approach required to solve problems involving the conservation of momentum in **two dimensions**, is to **resolve** the motion into components along **perpendicular axes** (e.g. the x and y-axis) and solve the resultant pair of problems in one dimension simultaneously.

For example, consider the collision of two billiard balls (assumed here to act as point masses), as shown in the diagram below:



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Firstly, consider the motion along the x-axis.

Total momentum before = Total momentum after

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

Where **m_1/m_2** is the mass, **v_{1x}/v_{2x}** is the initial velocity and **v'_{1x}/ v'_{2x}** is the final velocity in the x-direction.

In this example, ball 2 is **initially at rest** so $v_{2x} = 0$ and the above equation becomes:

$$m_1 v_{1x} = m_1 v'_{1x} + m_2 v'_{2x}$$

Ball 1 is initially moving in the x-direction, therefore we know that $v_{1x} = v_1$ (which is given in the diagram above).

$$m_1 v_1 = m_1 v'_{1x} + m_2 v'_{2x}$$

The final velocities (v'_1/ v'_2 - given in the diagram), can be **resolved** into components along the x and y-axis using trigonometry. *(Resolving vectors is explained fully in Topic 2.13).* Where the angles used are those given in the diagram.

By resolving the final velocities you get the following equation, which can be **used alongside the equation derived by considering the y-direction**, in order to solve for unknown values.

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2 \qquad \textbf{[1]}$$

Next, consider the motion along the y-axis.

Total momentum before = Total momentum after

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$$

Where **m_1/m_2** is the mass, **v_{1y}/v_{2y}** is the initial velocity and **v'_{1y}/ v'_{2y}** is the final velocity in the y-direction.

In this example, ball 2 is **initially at rest** so $v_{2y} = 0$ and ball 1 **initially moves along the x-axis** so $v_{1y} = 0$ and the above equation becomes:

$$0 = m_1 v'_{1y} + m_2 v'_{2y}$$

By resolving the final velocities you get the following equation, which can be **used alongside the equation derived by considering the x-direction**, in order to solve for unknown values.

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2 \qquad [2]$$

In the case where **m_1 = 5 kg, m_2 = 10 kg, v_1 = 5 ms^{-1}, v_2 = 0 ms^{-1}, θ_1 = 60° and θ_2 = 30°**, you can use the equations derived above to find the final velocities v'_1 and v'_2 as shown below.

Substitute in known values into equation [1]:

$$5 \times 5 = 5 \times v'_1 \cos 60 + 10 \times v'_2 \cos 30$$

$$25 = \tfrac{5}{2} \times v'_1 + 5\sqrt{3} \times v'_2 \qquad [3]$$

Substitute in known values into equation [2]:

$$0 = 5 \times v'_1 \sin 60 + 10 \times v'_2 \sin 30$$

$$0 = \tfrac{5\sqrt{3}}{2} \times v'_1 + 5 \times v'_2 \qquad [4]$$

Finally, solve the simultaneous equations.

- Multiply equation [3] by 6 to get: $150 = 15v'_1 + 30\sqrt{3} \times v'_2$ [5]
- Multiply equation [4] by 2√3 to get: $0 = 15v'_1 + 10\sqrt{3} \times v'_2$ [6]
- Subtract equation [6] from [5] to get: $150 = 20\sqrt{3} \times v'_2$
- Rearrange to get: $v'_2 = \frac{5\sqrt{3}}{2}\ ms^{-1}$
- Substitute v'_2 into equation [6] to get: $0 = 15v'_1 + 75$
- Rearrange to get: $v'_1 = -5\ ms^{-1}$

As you can see from the solutions for the final velocities of the balls, they are moving in **opposite** directions as expected.

4.3.85 - Elastic and inelastic collisions

There are two types of collisions:

- **Elastic** - where **both** momentum and kinetic energy are conserved
- **Inelastic** - where **only** momentum is conserved, while some of the kinetic energy is converted into other forms (e.g heat, sound, gravitational potential) and may be larger or smaller after a collision

If the objects which collide stick together after the collision, then this is an **inelastic collision**.

An **explosion** is another example of an inelastic collision as the kinetic energy after an explosion is **greater** than before the explosion.

4.3.86 - Kinetic energy of a non-relativistic particle

When considering a non-relativistic particle, which is one travelling below relativistic speeds (comparable to the speed of light), you can use the following formula to calculate its **kinetic energy (E_k)**:

$$E_k = \frac{p^2}{2m}$$

You must be able to **derive** the above formula using the formulas for kinetic energy and momentum:

$$E_k = \tfrac{1}{2}mv^2 \qquad\qquad p = mv$$

Firstly, rearrange the formula for momentum, so that its subject is velocity (v).

$$v = \tfrac{p}{m}$$

Next, substitute the equation above into the formula for kinetic energy.

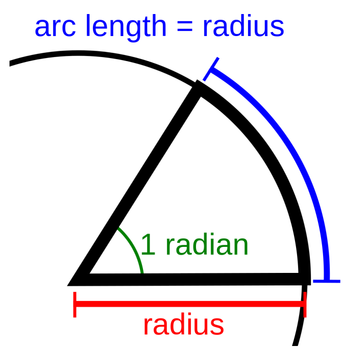
$$E_k = \tfrac{1}{2}m \times \tfrac{p^2}{m^2}$$

$$E_k = \frac{p^2}{2m}$$

4.3.87 - Angular displacement and radians

Angles can be measured in units called **radians**. One radian is defined as the angle in the sector of a circle when the arc length of that sector is equal to the radius of the circle, as shown in the diagram below.

Considering a complete circle, its arc length is $2\pi r$, dividing this by r, you get 2π which is the angle in radians of a full circle. From this you can convert any angle from **degrees to radians** by multiplying by $\frac{\pi}{180}$, and from **radians to degrees** by multiplying by $\frac{180}{\pi}$.



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Angular displacement (θ) is the **angle turned through** by an object in any given direction **in radians or degrees**.

4.3.88 - Angular velocity

Angular velocity (ω) is the angle an object moves through per unit time. It can be found by dividing the object's linear velocity (v) by the radius of the circular path it is travelling in (r):

$$\omega = \frac{v}{r}$$

Where **v** is the linear velocity and **r** is the radius of the circular path. This equation can be rearranged to:

$$v = \omega r$$

Angular velocity can also be found by dividing the angle in a circle in radians (2π) by the object's time period (T).

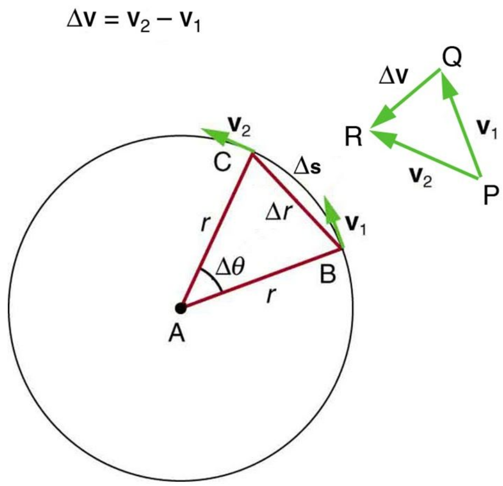
$$\omega = \frac{2\pi}{T}$$

This can be rearranged to:

$$T = \frac{2\pi}{\omega}$$

4.3.89 - Centripetal acceleration

Centripetal acceleration is experienced by objects moving in a circular path. You must be able to derive the formula for centripetal acceleration using **vector diagrams** as shown below.



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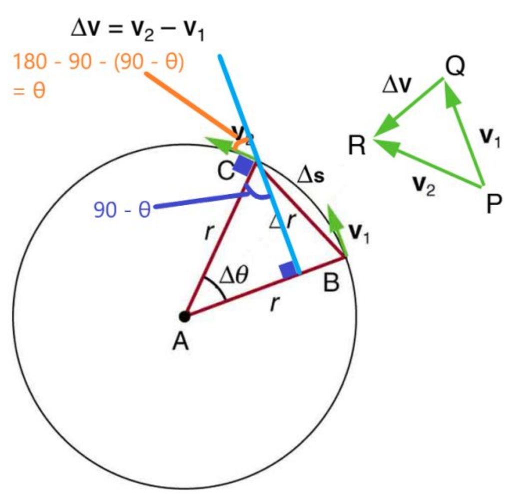
1. Consider an object moving at a constant speed **v**, in a circular path of radius **r**.
2. In the diagram above, the triangles formed as part of the circular path and by the velocity vectors are **similar**. This is because both have 2 sides of equal length (r/v) and it can be the angle between these equal sides is the **same**.
    - You can show that this angle is the same by drawing a line parallel to v_1, starting on the line AB, passing through the point C (this is the blue line on the diagram below).
    - As v_1 is perpendicular to AB, you have just made a right-angled triangle.
    - Using this right-angled triangle and the fact that all the angles in a triangle/on a straight line add up to 180°, you can show that the angle in the triangle formed by the velocity vectors is also θ.



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3. As these two triangles are **similar** (as shown above), you can write:

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

4. Rearrange to get **Δv** as the subject.

$$\Delta v = \frac{v}{r} \times \Delta s$$

5. Divide through by **Δt** as a = Δv/Δt.

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}$$

$$a = \frac{v}{r} \times \frac{\Delta s}{\Delta t}$$

6. **Simplify** the equation above using the fact that v = Δs/Δt.

$$a = \frac{v}{r} \times v$$

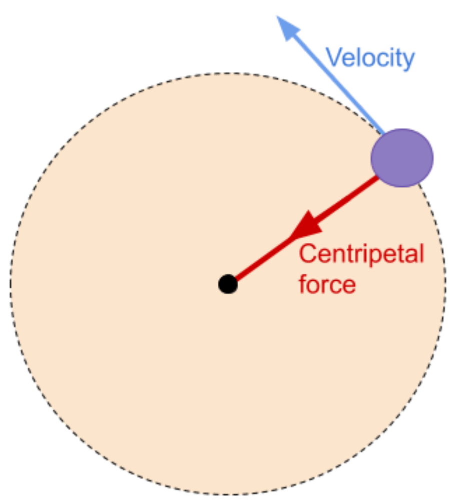
$$a = \frac{v^2}{r} = r\omega^2$$

(As $v = \omega r$)

Where **v** is linear speed, **r** is the radius of the path and **ω** is the angular velocity.

4.3.90 - Circular motion and centripetal force

An object moving in a circular path at constant speed has a constantly changing velocity as velocity has both magnitude and direction, therefore the object must be **accelerating** (this is known as centripetal acceleration). We know from Newton's first law that to accelerate, an object must experience a resultant force, therefore an object moving in a circle must experience a force, this is known as the **centripetal force**, and it always acts towards the centre of the circle.



A centripetal force is always required to **produce and maintain** circular motion.

4.3.91 - Centripetal force

Using **Newton's second law**, $F = ma$, we can derive the formula for **centripetal force (F)** from the formula for centripetal acceleration (above).

$$F = ma = \frac{mv^2}{r} = mr\omega^2$$

Where **m** is the mass of the object, **v** is linear speed, **r** is the radius of the path and **ω** is the angular velocity.